A SIMILARITY THEORY FOR THERMAL MASS FLOW SENSOR AND ITS GAS CONVERSION FACTORS

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Abstract - This paper proposes a similarity theory for the capillary thermal mass flow sensor. The theory expresses the sensor output, divided by the gas thermal conductivity, as a function of the Péclet number, i.e. (Re·Pr), of the flow inside of the sensor tube. The theory compares favorably with experimental data collected for a wide range of gases over a wide range of flow rate. The similarity model is useful because it not only describes the sensor output as a function of the flow rate, but also provides a method to scale the sensor output characteristic curve from gas to gas by using the thermal-physical properties alone. The similarity model is applicable in both the linear and the nonlinear range of the sensor. In the linear range, the model condenses into the conventional gas conversion factors widely in use by the thermal mass flow controller industry.

INTRODUCTION

The thermal mass flow sensor is an in-line laminar-flow device capable of continuously measuring the mass flow-rate in a flow path. The thermal mass flow sensor consists of a metal sensor tube (Fig. 1), over which are installed two temperature sensitive resistance elements serving as both the heating and the sensing device. When there is no fluid flow, the excitation current going through the two resistance elements heats up the sensor tube in a symmetrical manner. When there is flow, the fluid entering the sensor cools down the upstream resistor more than it does to the downstream resistor. In a typical mass flow sensor circuit, the two resistors form one branch of a Wheatstone bridge. The temperature difference between the two resistors is detected as the voltage imbalance in the bridge.

Since its inception mass flow sensors and controllers have been widely adopted for measuring and controlling the flow of gases in various chemical processes. This is mainly because unlike many other flow measurement devices that measure the volumetric flow rate, the mass flow sensors directly measure the mass (or molar) flow rate which is important for chemical processes. Also, most thermal mass flow sensors are made of corrosion resistant stainless-steel tubes which protect not only the sensor but also the purity of the chemical inside. This makes the thermal mass flow sensors attractive for ultra-clean processes that involve corrosive or sometimes even toxic substances. The type of gases used in thermal mass flow sensors ranges from simple inert gases such as He, N2, or Xe; large molecules such as SF6 or C4F8; and corrosive gases such as Cl2, HCl, BCl3, SiCl4, WF6 or Si(C2H5O)4. The flow-rate ranges from sub-sccm's when the sensor is used alone to hundreds of SLM's when the sensor is combined with a laminar flow bypass element installed in parallel.

It is not practical for mass flow sensor or controller (MFC) manufacturers to calibrate the MFCs directly with the process gas. On the one hand, the cost for stocking and testing with hundreds of specialty gases is prohibitive, even if the gases are non-corrosive. On the other hand, if the process gases of interest are corrosive, corrosion will contaminate the MFC during the calibration process and render it unacceptable for ultra-clean processes. The industry's standard approach has been to calibrate the MFC with an inert gas, such as nitrogen, at a flow rate equal to the desired process-gas flow-rate multiplied by a so-called gas correction factor or gas conversion factor.

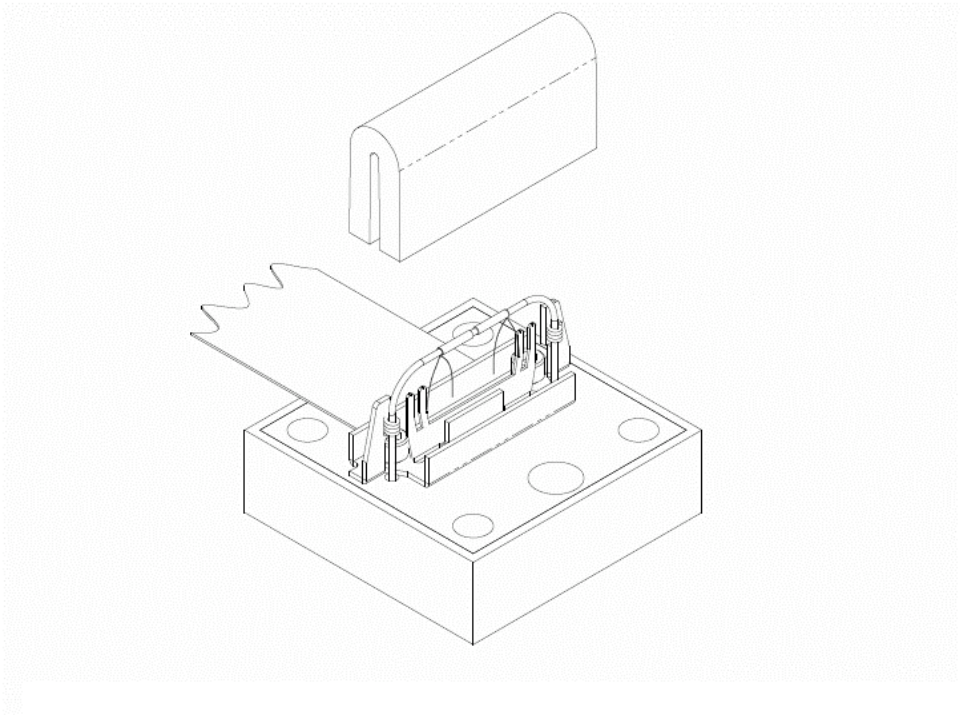


Fig. 1 The exploded view of a thermal mass flow sensor.

The gas conversion factor is generally identified with the ratio between the molar specific heat of the calibration gas and that of the process gas. The gas conversion factors are only approximate but are largely consistent with the industry's experience. For a small number of gases for which live-gas data are available, the conversion factors are adjusted to match the data to improve the MFC's accuracy. For most other gases where no data are available, the conversion factors are merely estimated from the gas molar specific heat, and the accuracy of the MFC is not guaranteed.

The theoretical basis for the gas conversion factors was never made very clear. Most literature quotes the first law of thermodynamics and argues that, for constant heat input, the mass flow rate is proportional to the heating rate divided by the gas specific-heat, i.e.:

$$\dot{m} = \frac{Const * Heating\ Rate}{c_p \cdot \Delta T} \tag{1}$$

While the first law of thermodynamics does suggest that, for an isolated system, the net energy input to be equal to the increase in the total internal energy, the gas in the thermal mass flow sensor tube is not in an isolated environment. First, there is gas flowing into and out-of the sensor tube. Second, there is heat conducted towards the two ends of the sensor; and, third, there is heat lost through the thermal insulation to the ambient. Besides, the gas temperature is seldom directly measured by the thermal mass-flow sensors. What is usually measured, i.e., the sensor output, comes from the difference between the voltage drop across the two resistance elements, which is determined by the upstream and the downstream tube temperature, with the later established by the gas flow in the tube. It is not easy to see how one may arrive at Eq.(1) for such a flowing system by using the first-law alone.

Regardless of the apparent lack of convincing justification, the simple conversion factor method has been adopted by the thermal mass flow sensor industry for many years, and it is largely consistent with MFC test experiences at least at relatively low flow rates. It was gradually realized, however, that the output of the thermal mass flow sensor was actually nonlinear. As will be shown in Fig. 3 below, the sensor output is a nonlinear function of the mass flow rate, and the sensor response curves for different gases behave quite differently. At low flow rates, in the so-called linear range, the sensor output does seem to be related to the flow rate by a constant factor. At higher flow rates, the curves depart more and more from straight lines and the constant conversion factor method is no longer satisfactory. The thermal mass flow sensors have a rather limited linear range. In fact the sensor output actually levels off at a certain flow rate and can even go down if one further increases the gas flow after the sensor signal levels off [1]. The nonlinearity in the thermal mass flow sensor has never been fully explored. Some attributed it to the sensor's design. Others mulled over the effects of gas properties such as the molecular weight, or the thermal conductivity or both. No general consensus was ever reached.

Beside nonlinearity, it was also learned that the gas conversion factors seem to vary somewhat from manufacturer to manufacturer. There was no good explanation of what is actually going on. In 1993 NIST, ORNL, and SEMATECH conducted a survey on the performance of thermal mass flow controllers. A critical evaluation of the results was given by Tison [2]. Numerous investigators studied the thermal mass flow sensor analytically [3] [4], or numerically [5] [6], but no one offered any clear explanations for how the gas conversion factor works in the linear range; why it breaks down in the nonlinear range; and moreover, how to scale the sensor flow once it breaks down. The present effort attempts to fill in this gap.

HEAT TRANSFER THEORY

Governing Equations

Consider the convective heat transfer of the fluid in a thermal mass flow sensor (Fig. 2). The axi-symmetric flow inside the sensor tube may be treated as a laminar flow with negligible pressure gradient and negligible viscous dissipation. For a homogeneous fluid with no chemical reaction and no internal heat source, the energy equation in cylindrical coordinate reads:

$$\rho u \frac{\partial i}{\partial x} + \rho v \frac{\partial i}{\partial r} - \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r k \frac{\partial t}{\partial r} \right) + \frac{\partial}{\partial x} \left(k \frac{\partial t}{\partial x} \right) \right] = 0 \qquad (2)$$

where i is the enthalpy, t is the temperature, ρ is the density, k is the thermal conductivity, and u and v are the axial and radial component of the gas velocity, respectively.

Let us assume that the laminar flow in the sensor tube has a fully-developed velocity profile for which the radial component of velocity v is identically zero. This is a reasonable assumption because the heated region of the thermal mass flow sensor typically occupies only a small portion of the entire tube length and is located far away from the tube entrance.

To further simplify the analysis, let us assume that the thermal conductivity k is a constant; and, the enthalpy is a simple function of temperature, i.e., $d i = c_p \, dt$, where the gas specific heat at constant pressure c_p is itself a constant. The energy equation becomes:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial t}{\partial r} \right) + \frac{\partial^2 t}{\partial x^2} = \frac{\rho u c_p}{k} \frac{\partial t}{\partial x} \qquad (3)$$

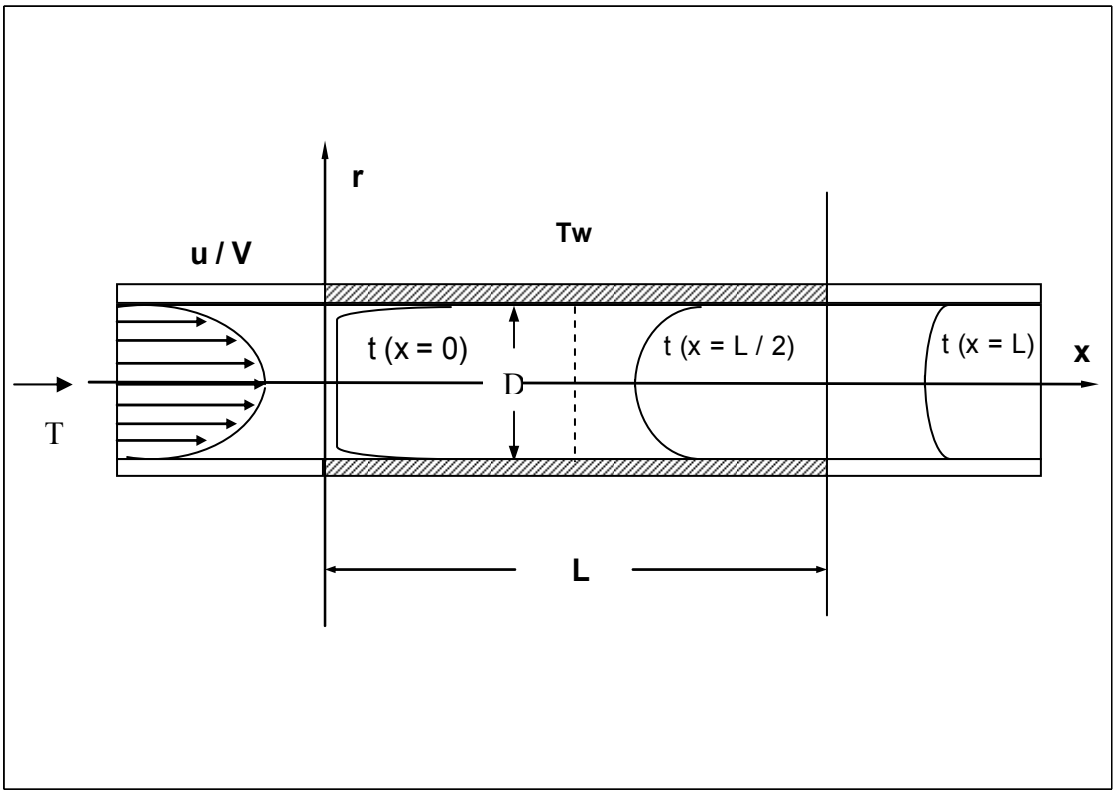


Fig. 2 Laminar flow heat transfer in the sensor tube.

Equation (3) may be written in the following dimensionless form [8]:

$$\frac{\partial^2\theta}{\partial\, r^{+2}} + \frac{1}{r^+}\frac{\partial\theta}{\partial r^+} = \frac{u^+}{2}\frac{\partial\theta}{\partial x^+} - \frac{1}{(\mathrm{Re}_D\cdot\mathrm{Pr})^2}\frac{\partial^2\theta}{\partial x^{+2}} \qquad (4)$$

where

$$\theta = \frac{(T_W - t\ )}{(T_W - T_0\ )},$$

$$x^+ = \frac{x/r_0}{\mathrm{Re}_D\,\mathrm{Pr}},$$

$$r^+ = r/r_0\ ,$$

$$u^+ = u/V\ ,$$

$$\mathrm{Re}_D = \frac{\rho\, V\, D}{\mu};\quad \mathrm{Pr} = \frac{\mu\, c_p}{k};\quad and,\quad \mathrm{Re}_D\cdot\mathrm{Pr} = \frac{\rho\, V\, D\, c_p}{k}\ .$$

In the above, r_0 is the radius and D is the inside diameter of the tube. T_0 is the temperature of the ambient. T_W is the tube inside wall temperature and is a constant for the constant temperature sensor under consideration. V is the reference flow velocity. V may be identified with the maximum in the sectional velocity profile.

The quantity $\mathrm{Re}_D \cdot \mathrm{Pr}$ above is frequently referred to as the Péclet number in the heat transfer literature. Note that the Reynolds number above may be written as

$$\mathrm{Re}_D = \frac{\rho\, VD}{\mu} = \left(\frac{4}{\pi}\right)\frac{\dot{m}}{\mu D}. \tag{5}$$

In a thermal mass flow sensor the mass flow-rate $\dot{m}$ does not vary along the length of the tube, thus the Reynolds number Re_D is a constant except for the variation of viscosity μ as a function of temperature. To further simplify the analysis, let us assume that μ is also a constant so that the coefficient $\mathrm{Re}_D \cdot \mathrm{Pr}$ in Eq(4) will be treated as a pure constant in the remaining analysis.

Eq.(4) governs the temperature distribution of the fluid in the sensor. For a sensor with a heated length L, Eq. (4) applies over the region

$$x^+ = 0 \sim \frac{2 \cdot \left(L/D\right)}{\mathrm{Re}_D\, \mathrm{Pr}} \tag{6}$$

Examining Eqs.(4) and (6) together, one may conclude that the temperature distribution in the fluid is governed by two non-dimensional quantities: $\mathrm{Re}_D \cdot \mathrm{Pr}$ and L/D. The formal solution to Eqs.(4) and (6) may thus be written as:

$$\theta = \left(\frac{T_W - t}{T_W - T_0}\right) = \theta\left\{x^+;\, r^+;\, \mathrm{Re}_D \cdot \mathrm{Pr};\, \frac{L}{D}\right\} \tag{7}$$

We will show that without explicitly solving the partial differential equation, one may still gain significant insight by examining the similarity property of the problem.

Thermal Mass Flow Sensor Similarity Model

For a constant temperature sensor, the sensor output is determined by the amount of heat generated by the electric current (provided by a feed back control loop) to offset the effect of gas flow and to keep the resistors at the fixed temperature. Here we are omitting the heat loss through the tube ends and the heat loss to the ambient through the thermal insulation on the ground that these heat losses do not change with gas flow, because the coils *are* being maintained at a fixed temperature. The heat that accounts for the gas flow must eventually be conducted into the fluid. The heat-flux into the fluid at the wall boundary per unit length of the tube is

$$\frac{dq}{dx} = 2\pi r_0 k\left(\frac{\partial t}{\partial r}\right)_{r=r_0} = -2\pi k\,(T_W - T_0)\left(\frac{\partial \theta}{\partial r^+}\right)_{r^+=1} \tag{8}$$

For a constant temperature thermal mass-flow sensor with two resistance elements symmetrically arranged on the tube, the output voltage S is proportional to the difference between the heat input to the upstream and the downstream resistance element, i.e.,

$$S = G \cdot (q_u - q_d) = \pi D G \cdot \left\{ \int_0^{L/2} k \left(\frac{dt}{dr} \right)_{r=r_0} dx - \int_{L/2}^{L} k \left(\frac{dt}{dr} \right)_{r=r_0} dx \right\},$$

where G is the electronic amplifier gain constant. In terms of the dimensionless variables:

$$\frac{S}{k} = -\pi D G \cdot (T_W - T_0) \cdot (\mathrm{Re}_D \cdot \mathrm{Pr}) \left\{ \int_0^{\frac{L}{D}\frac{1}{\mathrm{Re}_D \mathrm{Pr}}} \left(\frac{d\theta}{dr^+} \right)_{r^+=1} dx^+ - \int_{\frac{L}{D}\frac{1}{\mathrm{Re}_D \mathrm{Pr}}}^{\frac{L}{D}\frac{2}{\mathrm{Re}_D \mathrm{Pr}}} \left(\frac{d\theta}{dr^+} \right)_{r^+=1} dx^+ \right\} \qquad (9)$$

The quantity in the curly bracket is a function of $\mathrm{Re}_D \cdot \mathrm{Pr}$ and L/D only. Since the quantities G, D, L/D, and $(T_w - T_0)$ are all fixed sensor design constants, Eq.(9) suggests that, for sensors of a given design, the quantity S/k is a universal function of $\mathrm{Re}_D \cdot \mathrm{Pr}$, the Péclet number of the flow. Denote this universal function W, we obtain the following similarity model for thermal mass flow sensor:

$$\frac{S}{k} \propto W\{\mathrm{Re}_D \cdot \mathrm{Pr}\} = W\left\{ \frac{\rho V D c_p}{k} \right\} \qquad (10)$$

Eq.(10) suggests that one should correlate the sensor output characteristic curves by using the similarity variables S/k and $\mathrm{Re}_D \cdot \mathrm{Pr}$. While Eq.(9) does suggest that the sensor characteristic curve may vary depending on the sensor's design, Eq.(10) suggests that the similarity concept should hold regardless.

EXPERIMENTAL DATA

Actual sensor data were collected from constant-temperature thermal mass flow sensors with a tube ID of 0.0135 inches and a heated length of 0.5 inches. The heated portion of the tube was covered by two resistance coils symmetrically located and maintained at 100 °C. Fig.3 shows the sensor output electric signal plotted against the flow rate for various gases. As shown in Fig. 3, significant nonlinearity accompanies the data for several gases such as CHF3, CF4, and SF6.

To verify the similarity theory, the data in Fig. 3 are re-plotted in Fig. 4 by using the non-dimensional quantities suggested above. Since L/D is a constant for the sensors we tested, instead of Re_DPr, the quantity $Re_LPr = Re_DPr(L/D)$ is used in the horizontal axis of the plot. Gas property constants at an average temperature of 85 °C were used for calculating the dimensionless quantities. As shown in Fig. 4, the $S/k \sim Re_LPr$ curves from all 8 different gases collapsed into a narrow band, suggesting that the similarity theory works quite well.

Some of the curves in Fig. 4 do noticeably deviate from the mean at the high end. The departure of the curves may be attributed to: (1) the various simplifying assumptions made in deriving the similarity model; and, (2) the uncertainty in the gas thermal-physical property data. For practical applications, we tentatively assume that the deviations to be entirely due to the uncertainties in the existing gas properties, which are then corrected by adjusting the gas specific heat by a multiplier e_c and the thermal conductivity by a multiplier e_k. To determine the values of e_c and e_k, live-gas tests are conducted to cover the range of sensor flow-rate of interest. The process gas $S/(k \cdot e_k)$ versus $(Re_LPr \cdot e_c/e_k)$ curves are then plotted and compared against the curve for the calibration gas, in this

case argon, for which the gas properties are assumed to be exact (with e_c and e_k both equal to 1.0). The process gas property correction constants e_c and e_k are adjusted until the r.m.s. error between the two curves is minimized. After doing so for all 8 gases, the correlation is much improved, as shown in Fig. 5. It turns out that the values of the correction constants do not vary significantly from sensor to sensor for sensors of the same design. In addition, the gas property correction constants e_c and e_k for most gases tested all fall pretty close to 1.0, as shown in Table 1. This suggests that the similarity theory is physically sound. A short list of semi-conductor process gases other than those in Table 1 were also tested and compared favorably against the similarity model. For examples, the results obtained from BCl3, Cl2,HCl, HBr, SiCl4, C4F6, and C4F8, etc. are all affirmative.

Fig. 3 Sensor Response to Different Gas Flow

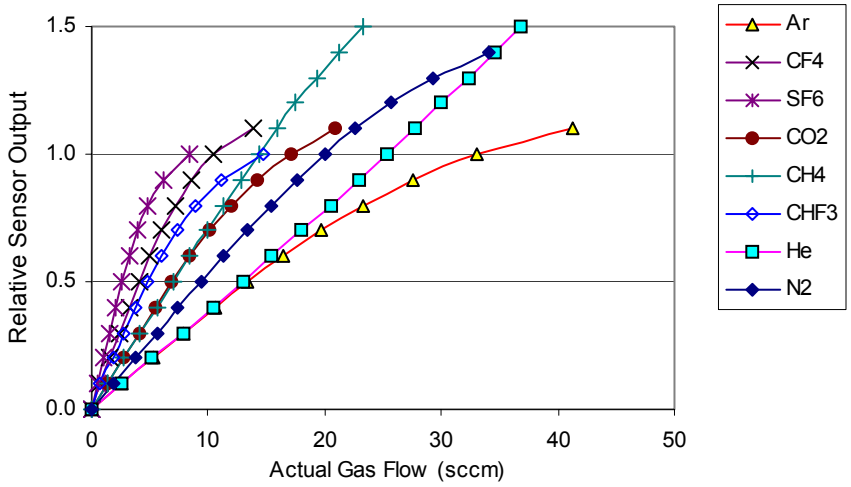


Fig. 4 Sensor Response Expressed in Similarity Variables

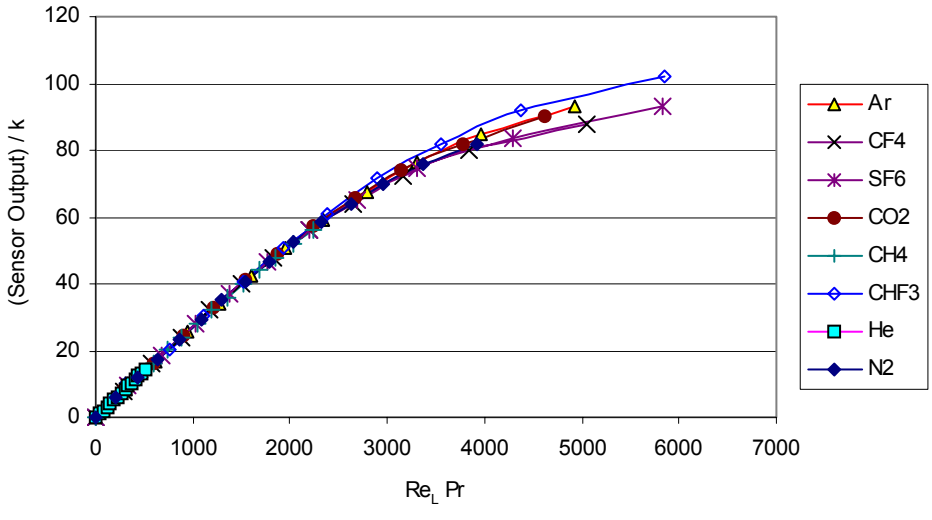
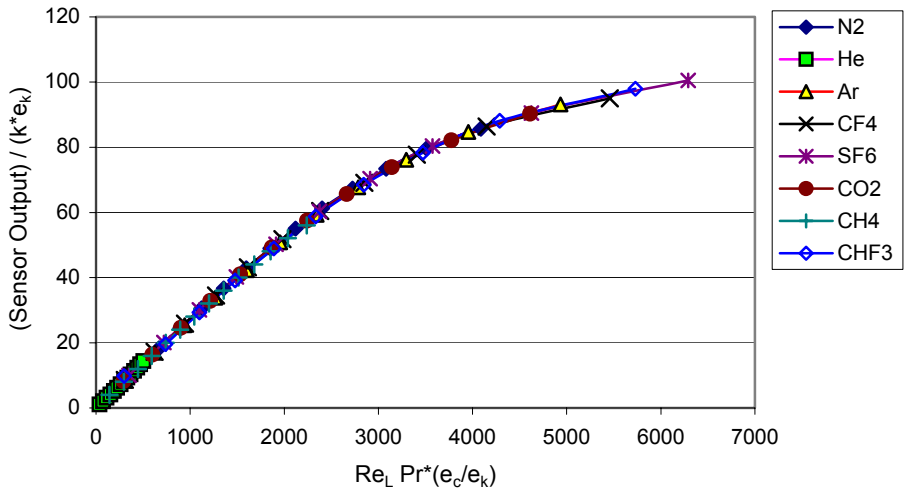


Fig. 5 Sensor Response Expressed in Similarity Variables
(Adjusted Gas Property Constants)



THE GAS CONVERSION FACTOR

One of the interesting results from the similarity theory is its implication on the gas conversion factors. Let us examine the similarity model Eq. (10), here rewritten as

$$S = k \cdot W(\mathrm{Re}_D \cdot \mathrm{Pr}) = k \cdot W(\frac{\dot{m} C_p}{k D}) \qquad (11)$$

Denote $Re_D Pr$ as Φ and take the derivative of Eq. (11) with respect to $\dot{m}$. We obtain

$$\frac{d S}{d \dot{m}} = k \cdot W'(\Phi) \cdot \frac{C_p}{kD} = \frac{C_p}{D} \cdot W'(\Phi) \qquad (12)$$

The left-hand side of Eq. (12) represents the sensitivity of the sensor to the mass flow rate (for example in the unit of volts per sccm). The right-hand side is proportional to the specific heat of the gas, multiplied by the derivative of the sensor characteristic function W with respect to the similarity parameter $Re_D Pr$. Taking the ratio between the sensitivity of the sensor to the process gas (PG) to the sensitivity to nitrogen (N2), we find:

$$\frac{[dS/d\dot{m}]_{PG}}{[dS/d\dot{m}]_{N2}} = \frac{(C_p)_{PG} \cdot W'(\Phi_{PG})}{(C_p)_{N2} \cdot W'(\Phi_{N2})}.$$

For a given sensor, W is a universal function for all gases. Suppose one operates the sensor at the same value Φ so that $\Phi_{PG} = \Phi_{N2}$ and $W'(\Phi_{PG}) = W'(\Phi_{N2})$, the above relation becomes:

$$\frac{[dS/d\dot{m}]_{PG}}{[dS/d\dot{m}]_{N2}} = \frac{(C_p)_{PG}}{(C_p)_{N2}}. \qquad (13)$$

Since under the empirical conversion factor method the sensors operate in the linear range, where the slope $dS/d\dot{m}$ is simply equal to $S/\dot{m}$. Eq.(4.4) reduces to

$$\frac{[S/\dot{m}]_{PG}}{[S/\dot{m}]_{N2}} = \frac{(C_p)_{PG}}{(C_p)_{N2}}. \tag{14}$$

At the same sensor output voltage S we therefore have the following relationship:

$$\frac{[\dot{m}]_{N2}}{[\dot{m}]_{PG}} = \frac{(C_p)_{PG}}{(C_p)_{N2}} \tag{15}$$

Since $\dot{m} = \rho\dot{Q}$, where $\dot{Q}$ is the gas volumetric flow rate, *for a perfect gas* the above relation becomes:

$$\frac{\rho_{N2}\ \dot{Q}_{N2}}{\rho_{PG}\ \dot{Q}_{PG}} = \frac{[PM_{PG}/\hat{R}T]\,\dot{Q}_{N2}}{[PM_{N2}/\hat{R}T]\,\dot{Q}_{PG}} = \frac{(C_p)_{PG}}{(C_p)_{N2}} \tag{16}$$

where P is the pressure, M is the molecular weight, and $\hat{R}$ is the universal gas constant. Thus under the STP conditions the standard flow rate of the process gas is related to the standard flow rate of nitrogen as follows:

$$\frac{\dot{Q}_{N2}}{\dot{Q}_{PG}} = \frac{M_{N2}\cdot(C_p)_{PG}}{M_{PG}\cdot(C_p)_{N2}} = \frac{(\hat{C}_p)_{PG}}{(\hat{C}_p)_{N2}} \tag{17}$$

, where $\hat{C}p$ is the *molar* specific heat of the gas. Eq.(17) is consistent with the definition of the conventional gas conversion factors. We have therefore recovered the conventional conversion factor relationship for the thermal mass flow sensor by using heat transfer theory and similarity argument.

CONCLUSIONS

A similarity theory for the thermal mass flow sensor is presented. The similarity theory serves several purposes. It provides a method to calibrate the sensor with a single calibration gas. Once calibrated, it is capable of predicting the process gas flow-rate, as long as the accurate thermal physical properties are available. It also provides a method to scale the sensor response curves from gas to gas in both the linear and the nonlinear range, allowing the sensor to be used for a wider range of gases and over a wider range of flow-rates. This feature is extremely useful for multi-gas and multi-flow configurable MFC's.

Based on laminar flow heat transfer theory, the similarity model offered a new explanation for the conventional gas-conversion factors. It not only explained how the conventional conversion factor works in the linear region but also suggested how one should scale the flow from gas to gas once the conventional conversion factors break down. In a sense the similarity model fills a void in the existing thermal mass-flow sensor theories.

The general agreement between the similarity model and test data is good. Deviations of the data from theory is attributed to the uncertainties in the gas thermal-physical property constants and to the various approximations made in deriving the model. Further investigations will be needed to re-examine the various assumptions made during model derivation for future improvement.

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Table 1 Values of e_c and e_k constants used in plotting Fig. 5.

Gas	**e_c**	**e_k**
N2	0.99	0.95
He	1.00	1.00
Ar	1.00	1.00
CH4	1.00	1.00
CO2	1.00	1.00
CF4	1.00	0.93
CHF3	0.98	1.02
SF6	1.00	0.93